Verification Test Number

102

Scope

- Verify beam detailing calculations
- Verify beam design resistance calculations
- Verify beam rotational ductility calculations
- Verify beam general and seismic checks

Files Used

Example 1 files

Procedure

Beam: B1, Story 1, COMB2X MAX

Reinforcement

	I-End	Mid-span	J-End
Top	3Y16	3Y16	3Y16
Area	603 mm^2	603 mm^2	603 mm^2
Bottom	3Y16	3Y16	3Y16
Area	603 mm^2	603 mm^2	603 mm^2

Table 1. Beam reinforcement at all regions.

The following calculations are typical and apply to all beam regions, since they have the same properties.

Dimensions

Width

$$b = 250\,mm$$

Depth

$$h = 500\,mm$$

Cover to stirrups

$$cover = 25\,mm$$

Materials

Concrete nominal strength

$$f_c = 40\,MPa$$

Partial factor for concrete (Seismic)

$$\gamma_c = 1.50$$

Concrete design strength

$$f_{cd} = 26.7\,MPa$$

Concrete ultimate strain

$$\varepsilon_u = 0.0035$$

Concrete peak strain

$$\varepsilon_{c1} = 0.0022$$

Concrete elastic modulus

$$E_c = 35\,GPa$$

Reinforcement steel yield stress

$$f_y = 460\,MPa$$

Partial factor for reinforcement steel (Seismic)

$$\gamma_s = 1.15$$

Reinforcement steel design strength

$$f_{yd} = 400\,MPa$$

Reinforcement steel elastic modulus

$$E_s = 200\,GPa$$

Reinforcement

Top reinforcement diameter

$$d_{rt} = 16\,mm$$

Number of top reinforcement bars

$$n_{rt} = 3$$

Top reinforcement area

$$A_{rt} = 603\,mm^2$$

Bottom reinforcement diameter

$$d_{rb} = 16\,mm$$

Number of bottom reinforcement bars

$$n_{rb} = 3$$

Bottom reinforcement area

$$A_{rb} = 603\,mm^2$$

Shear reinforcement diameter

$$d_{rv} = 8\,mm$$

Number of shear reinforcement stirrups

$$n_{rv} = 1$$

Shear reinforcement spacing

$$s_{rv} = 100\,mm$$

Total area of shear reinforcement

$$A_v = 100\,mm^2$$

Depth to tension reinforcement centre

$$d_t = h - cover - d_{rv} - \frac{d_{rb}}{2}$$
$$= 500\,mm - 25\,mm - 8\,mm - \frac{16\,mm}{2}$$
$$= 459\,mm$$

Depth to compression reinforcement centre

$$d_c = cover + d_{rv} + \frac{d_{rt}}{2}$$
$$= 25\,mm + 8\,mm + \frac{16\,mm}{2}$$
$$= 41\,mm$$

Detailing

Top reinforcement spacing

$$s_{rt} = \frac{b - 2 \cdot cover - 2 \cdot d_{rv} - d_{rt}}{(n_{rt} - 1)} = \frac{250\,mm - 2 \cdot 25\,mm - 2 \cdot 8\,mm - 16\,mm}{(3 - 1)}$$
$$= 84\,mm$$

Bottom reinforcement spacing

$$s_{rb} = \frac{b - 2 \cdot cover - 2 \cdot d_{rv} - d_{rb}}{(n_{rb} - 1)} = \frac{250\,mm - 2 \cdot 25\,mm - 2 \cdot 8\,mm - 16\,mm}{(3 - 1)}$$
$$= 84\,mm$$

Minimum reinforcement spacing

$s_{\min} = 50\,mm$ Check OK

Maximum reinforcement spacing

$s_{\max} = 150\,mm$ Check OK

Top reinforcement ratio

$$\rho' = \frac{A_{s'}}{b \cdot d_t} = \frac{603\,mm^2}{250\,mm \cdot 459\,mm} = 0.0053 = 0.53\%$$

Bottom reinforcement ratio

$$\rho = \frac{A_s}{b \cdot d_t} = \frac{603\,mm^2}{250\,mm \cdot 459\,mm} = 0.0053 = 0.53\%$$

Minimum reinforcement ratio

$\rho_{\min} = 0.13\%$ Check OK

Maximum reinforcement ratio

$\rho_{\max} = 4.00\%$ Check OK

Neutral Axis Depth at Ultimate Resistance

Neutral axis depth

$$c = 42\,mm$$

Concrete stress-block parameters

$$\beta = 0.380$$
$$\beta_1 = 2 \cdot \beta = 0.760$$

Concrete stress-block actual compressive force

$$F_c = 0.85 \cdot \beta_1 \cdot c \cdot f_c \cdot b = 0.85 \cdot 0.760 \cdot 0.042\,m \cdot 40E+3\,\frac{KN}{m^2} \cdot 0.250\,m$$
$$= 271\,KN$$

Tension reinforcement strain

$$\varepsilon_s = \varepsilon_{cu} \cdot \frac{(d_t - c)}{c} = 0.0035 \cdot \frac{(459\,mm - 42mm)}{42\,mm}$$
$$= 0.030$$

Tension reinforcement actual stress

$$f_s = \varepsilon_u \cdot E_s \cdot \frac{(d_t - c)}{c} = 0.0035 \cdot 200E+3\,\frac{N}{mm^2} \cdot \frac{(459\,mm - 42mm)}{42\,mm}$$
$$= 6{,}950\,\frac{N}{mm^2} \le f_y$$
$$f_s = f_y = 460\,\frac{N}{mm^2}$$

Tension reinforcement actual force

$$F_s = A_s \cdot f_y = 603\,mm^2 \cdot 460\,\frac{N}{mm^2} = 278\,KN$$

Compression reinforcement actual stress

$$f_{s'} = \varepsilon_u \cdot E_s \cdot \frac{(c - d_c)}{c} = 0.0035 \cdot 200E+3\,\frac{N}{mm^2} \cdot \frac{(42\,mm - 41\,mm)}{42\,mm}$$
$$= 17\,\frac{N}{mm^2} \le f_y$$

Compression reinforcement actual force

$$F_{s'} = A_{s'} \cdot f_{s'} = 603\,mm^2 \cdot 17\,\frac{N}{mm^2} = 10\,KN$$

Force unbalance

$$\text{Force Unbalance} = \frac{F_c + F_{s'} - F_s}{F_c + F_{s'}} = \frac{271\,KN + 10\,KN - 278\,KN}{271\,KN + 10\,KN}$$
$$= 0.010 = 1.0\%$$

Design Bending Moment Resistance
Neutral axis depth

$$c = 42\,mm$$

Concrete stress-block design compressive force

$$F_{cd} = 0.85 \cdot \beta_1 \cdot c \cdot f_{cd} \cdot b = 0.85 \cdot 0.760 \cdot 0.042\,m \cdot 26.7E+3\,\frac{KN}{m^2} \cdot 0.250\,m$$
$$= 181\,KN$$

Tension reinforcement design stress

$$f_{sd} = \varepsilon_u \cdot E_s \cdot \frac{(d_t - c)}{c} = 0.0035 \cdot 200E+3\,\frac{N}{mm^2} \cdot \frac{(459\,mm - 42\,mm)}{42\,mm}$$
$$= 6{,}950\,\frac{N}{mm^2} \le f_{yd} = 400\,\frac{N}{mm^2}$$
$$f_{sd} = f_{yd} = 400\,\frac{N}{mm^2}$$

Tension reinforcement design force

$$F_{sd} = A_s \cdot f_{sd} = 603\,mm^2 \cdot 400\,\frac{N}{mm^2} = 241\,KN$$

Compression reinforcement design stress

$$f_{sd'} = \varepsilon_u \cdot E_s \cdot \frac{(c - d_c)}{c} = 0.0035 \cdot 200E+3\,\frac{N}{mm^2} \cdot \frac{(42\,mm - 41\,mm)}{42\,mm}$$
$$= 17\,\frac{N}{mm^2} \leq f_{yd} = 400\,\frac{N}{mm^2}$$

Compression reinforcement design force

$$F_{sd'} = A_{s'} \cdot f_{sd'} = 603\,mm^2 \cdot 17\,\frac{N}{mm^2} = 10\,KN$$

Bending moment design resistance

$$M_{Rd} = F_{sd'} \cdot (c - d_c) + F_{cd} \cdot (c - \beta \cdot c) + F_{sd} \cdot (d_t - c)$$
$$= 10\,KN \cdot (0.042\,m - 0.041\,m) + 181\,KN \cdot (0.042\,m - 0.380 \cdot 0.042\,m) +$$
$$+ 241\,KN \cdot (0.459\,m - 0.042\,m)$$
$$= 105\,KNm$$

Maximum Reinforcement for Tensile Failure

The tension reinforcement strain is higher than 0.005 which is the minimum allowable by ACI-318, clause 10.3.4, for a section to be classified as ‘Tension-Controlled’, therefore tensile failure is ensured.

Neutral Axis Depth at First Yield

Neutral axis depth

$$c_{yld} = 96.5\,mm$$

Concrete strain and stress

$$\varepsilon_c = \frac{f_y}{E_s} \cdot \frac{d_{seg}}{(d_{ty} - c_{yld})} \leq f_y$$

Concrete stress-block actual compressive force

Segment	Length (mm)	Distance from NA (mm)	ε_c	f_c (N/mm^2)	F_c (N)
1	24.125	12.0625	0.0001	2.6673	16,087
2	24.125	36.1875	0.0002	7.6215	45,967
3	24.125	60.3125	0.0004	12.0894	72,914
4	24.125	84.4375	0.0005	16.0941	97,068
				Total	232,036

Tension reinforcement actual force

$$F_s = A_s \cdot f_y = 603\, mm^2 \cdot 460\ N/mm^2 = 277\ KN$$

Compression reinforcement actual stress

$$f_{s'} = f_y \cdot \frac{(c_{yld} - d_c)}{(d_t - c_{yld})} = 460E+3\ N/mm^2 \cdot \frac{(96.5\, mm - 41\, mm)}{(459\, mm - 96.5\, mm)}$$
$$= 70\ N/mm^2 \leq f_y$$

Compression reinforcement actual force

$$F_{s'} = A_{s'} \cdot f_{s'} = 603\, mm^2 \cdot 70\ N/mm^2 = 42\ KN$$

Force unbalance

$$\text{Force Unbalance} = \frac{F_c + F_{s'} - F_s}{F_c + F_{s'}} = \frac{232\ KN + 42\ KN - 277\ KN}{232\ KN + 42\ KN}$$
$$= 0.010 = 1.0\%$$

Rotational Ductility

Curvature at first yield

$$\phi_y = \frac{f_y}{E_s} \cdot \frac{1}{d_t - c_{yld}} = \frac{460\ N/mm^2}{200E+3\ N/mm^2} \cdot \frac{1}{459\ mm - 96.5\ mm} = 0.00000634$$

Ultimate curvature

$$\phi_u = \frac{\varepsilon_u}{c_{ult}} = \frac{0.0035}{42\ mm} = 0.0000833 \frac{1}{mm}$$

where c_{ult} is the neutral axis depth at ultimate resistance

Rotational ductility

$$\mu_\phi = \frac{\phi_u}{\phi_y} = \frac{0.0000833 \frac{1}{mm}}{0.00000634 \frac{1}{mm}} = 13.13$$

Design Shear Resistance

Lever arm

$$z = d_t - d_c = 459\, mm - 41\, mm = 418\, mm$$

Design shear resistance contribution of reinforcement

$$V_s = \frac{A_v \cdot z \cdot f_{yd}}{s_{rv}} = \frac{100\, mm^2 \cdot 418\, mm \cdot 400\ N/mm^2}{100\, mm}$$
$$= 167\ KN$$

Total design shear resistance

$$V_t = V_s = 167\ KN$$

Maximum shear resistance

$$V_{\max} = \frac{b \cdot z \cdot 0.6 \cdot f_{cd}}{2} = \frac{250\,mm \cdot 418\,mm \cdot 0.6 \cdot 20\ N/mm^2}{2}$$
$$= 627\ KN$$

Check OK

Bending Moment Resistance Ratio Checks

Design negative bending moment resistance at i-end support

$$M_{-ve,\,i-end} = -105\ KNm$$

Required positive bending moment resistance at i-end support

$$M_{+ve,\ i\text{-}end} = \frac{\left|M_{-ve,\ i\text{-}end}\right|}{3} = \frac{105\ KNm}{3} = 35\ KNm$$

Design negative bending moment resistance at j-end support

$$M_{-ve,\ j-end} = -105\ KNm$$

Required positive bending moment resistance at j-end support

$$M_{+ve,\ j\text{-}end} = \frac{\left|M_{-ve,\ j\text{-}end}\right|}{3} = \frac{105\ KNm}{3} = 35\ KNm$$

Required negative bending moment resistance at mid-span

$$M_{-ve,\ mid\text{-}span} = \frac{Min\left(\left|M_{-ve,\ i\text{-}end}\right|, \left|M_{-ve,\ j\text{-}end}\right|\right)}{5} = -\frac{105\ KNm}{5} = -21\ KNm$$

Maximum Design Shear Force Checks

The following calculations are based on the results of the RC-PADD model which considers the intermediate reinforcement.

Design bending moment resistance of column C1 above i-end

$$M_{Rd2,\ i-end,\ above} = 103\ KNm$$
$$M_{Rd3,\ i-end,\ above} = 232\ KNm$$

Design bending moment resistance of column C1 below i-end

$$M_{Rd2,\ i-end,\ below} = 110\ KNm$$
$$M_{Rd3,\ i-end,\ below} = 245\ KNm$$

Sum of design bending moment resistance of columns i-end

$$\sum M_{Rd2,\ i-end} = 213\ KNm$$
$$\sum M_{Rd3,\ i-end} = 477\ KNm$$

Design bending moment resistance of column C2 above j-end

$$M_{Rd2,\ j-end,\ above} = 108\ KNm$$
$$M_{Rd3,\ j-end,\ above} = 243\ KNm$$

Design bending moment resistance of column C2 below j-end

$$M_{Rd2,\ j-end,\ below} = 120\ KNm$$
$$M_{Rd3,\ j-end,\ below} = 263\ KNm$$

Sum of design bending moment resistance of columns at j-end

$$\sum M_{Rd2,\ j-end} = 228\ KNm$$
$$\sum M_{Rd3,\ j-end} = 506\ KNm$$

Design positive bending moment resistance of connected beam B2 at j-end

$$M_{Rdpve,\ i-end} = 105\ KNm$$

Design negative bending moment resistance of connected beam B2 at j-end

$$M_{Rdnve,\ i-end} = -105\ KNm$$

Sum of design bending moment resistance of beams at j-end

$$\sum M_{Rd,\ j-end} = 210\ KNm$$

Beam overstrength factor

$$\gamma_{Rd,\ beam} = 1.0$$

Maximum bending moment that can develop at i-end

$$M_{dpve,\ i-end} = \gamma_{Rd,\ beam} \cdot M_{Rdpve,\ i-end} \cdot Min\left(1, \frac{\sum_{Columns} M_{Rd3,\ i\text{-}end}}{\sum_{Beams} M_{Rd,\ i\text{-}end}}\right)$$
$$= 1.0 \cdot 105\ KNm \cdot Min\left(1, \frac{477\ KNm}{105\ KNm}\right)$$
$$= 105\ KNm$$

$$M_{\mathrm{dnve,\ i-end}} = -105\ KNm$$

Maximum bending moment that can develop at j-end

$$M_{dpve,\,j-end} = \gamma_{Rd,\,beam} \cdot M_{Rdpve,\,j-end} \cdot Min\left(1, \frac{\sum_{Columns} M_{Rd3,\,j\text{-}end}}{\sum_{Beams} M_{Rd,\,j\text{-}end}}\right)$$

$$= 1.0 \cdot 105\ KNm \cdot Min\left(1, \frac{506\ KNm}{210\ KNm}\right)$$

$$= 105\ KNm$$

$$M_{dnve,\,j-end} = -105\ KNm$$

Design shear force at critical region due to gravity loads

$$V_{Ed,gr} = 1.0 \cdot DL + 0.3 \cdot LL = 1.0 \cdot 43\ KN + 0.3 \cdot 8\ KN = 45\ KN$$

Maximum shear force that can develop on the beams is

$$V_{Ed\,\max} = \frac{\left(\left|M_{dpve,\,i-end}\right| + \left|M_{dnve,\,j-end}\right|\right)}{L} + V_{Ed,gr}$$

$$= \frac{(105\ KNm + 105\ KNm)}{6.0\,m} + 45\ KN = 80\ KN$$